$-f(x_1, y_1) = h\frac{\partial f_1}{\partial x_1} + k\frac{\partial f_1}{\partial y_1}$

$-g(x_1, y_1) = h\frac{\partial g}{\partial x_1} + k\frac{\partial g_1}{\partial y_1}$

$0.19 = h(5.8) + k \quad — (5)$

$-1.56 = h + 4.8k \quad — (6)$

$h = -1.66 - 4.8k$

Then

$0.19 = (-1.66 - 4.8k)5.8 + k$

$0.19 = 9.62 - 27.84k + k$

$k = 0.351$

$\Rightarrow h = -1.66 - 4.8 \times 0.351$

$h = -3.34$

Now

The IInd approx^n can be written as

$x_2 = 2.98 - 3.34 = -0.36$

$y_2 = 2.47 + 0.35 = 2.82$

Gauss elimination method

Eigen value & Eigen vector →

If an algebric eq^n

$|A - \lambda I| = 0$

Then the corresponding roots of this eq^n is known as characteristic roots or eigen value of the algebric eq^n. Also if the algebric eq^n represented in the form of

$|A - \lambda I| x = 0$

Then such eq^n is known as characteristic eq^n & its roots is known as characteristic roots.

To calculate the eigen value of any matrix Ist of all we maltiply the matrix by the diagonal matrix with its transpose.

The prop. of eigen values are

1. The sum of the diagonal element of any matrix represent the trace of the matrix

2. Any matrix contains eigen value corresponding to its order & transpose of such matrix also represent same eigen value.

3. If any matrix is operated on any vector then the resultant value & corresponding vector also represent is eigen value.

$$\frac{x}{-2} = \frac{y}{2-1} = \frac{z}{2}$$

when $\lambda = 5$

$$\begin{vmatrix} -2 & 1 & 4 \\ 0 & -3 & 6 \\ 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix} = 0$$

$-2x + y + 4z = 0$

$0x - 3y + 6z = 0$

$$\frac{x}{6+12} = \frac{y}{1\cdot 2} = \frac{z}{6}$$

$$\frac{x}{3} = \frac{y}{2} = \frac{z}{1}$$

i.e $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

Dignodisation of matrix If A is any matrix of order n then $P^{-1}AP$ is known as dignolisation of matrix i.e

$$\boxed{P^{-1}AP = D}$$

for this let us consider a matrix A such that

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

whose eigen values are $\lambda_1, \lambda_2, \lambda_3$ & eigen vectors are X_1, X_2, X_3 such that

$X_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$, $X_2 = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$ & $X_3 = \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix}$

Then from the defn of eigen vectors corresponding to the eigen value λ_1 we get

$$|A - \lambda_1 I| X_1 = 0$$

Then $\begin{bmatrix} a_1 - \lambda_1 & b_1 & c_1 \\ a_2 & b_2 - \lambda_1 & c_2 \\ a_3 & b_3 & c_3 - \lambda_1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = 0$

also $(a_1 - \lambda_1)x_1 + b y_1 + c_1 z_1 = 0$

$a_2 x_1 + (b_2 - \lambda_1) y_1 + c_2 z_1 = 0$

$a_3 x_1 + b_3 y_1 + (c_3 - \lambda_1) z_1 = 0$

also
$$\left.\begin{aligned} a_1 x_1 + b_1 y_1 + c_1 z_1 &= \lambda_1 x_1 \\ a_2 x_1 + b_2 y_1 + c_2 z_1 &= \lambda_1 y_1 \\ a_3 x_1 + b_3 y_1 + c_3 z_1 &= \lambda_1 z_1 \end{aligned}\right\} \text{--- (1)}$$

rlly, for λ_2 & λ_3 the above eqn can be represented as

$$\left.\begin{aligned} a_1 x_2 + b_1 y_2 + c_1 z_2 &= \lambda_2 x_2 \\ a_2 x_2 + b_2 y_2 + c_2 z_2 &= \lambda_2 y_2 \\ a_3 x_2 + b_3 y_2 + c_3 z_2 &= \lambda_2 z_2 \end{aligned}\right\} \text{--- (2)}$$

$$\left.\begin{aligned} a_1 x_3 + b_1 y_3 + c_1 z_3 &= \lambda_3 x_3 \\ a_2 x_3 + b_2 y_3 + c_2 z_3 &= \lambda_3 y_3 \\ a_3 x_3 + b_3 y_3 + c_3 z_3 &= \lambda_3 z_3 \end{aligned}\right\} \text{--- (3)}$$

Eg → find the eigen value & eigen vector of the matrix.

$$A = \begin{vmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{vmatrix}$$

Sol^n from the cond^n of eigen value we know that

$$A - \lambda I = 0$$

writing down

$$\begin{vmatrix} 1 & 0 & 1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{vmatrix} - \lambda \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1-\lambda & 0 & 1 \\ 1 & 2-\lambda & 1 \\ 2 & 2 & 3-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)\,[6 - 2\lambda - 3\lambda + \lambda^2 - 2] - 1\,[2 - 4 + 2\lambda] = 0$$

$$(1-\lambda)\,[\lambda^2 - 5\lambda + 6] + (2\lambda - 2) = 0$$

$$(\lambda - 1)\,[5\lambda - \lambda^2 - 6] = 0$$

$$\lambda = 1$$

$$\lambda^2 - 5\lambda + 6 = 0$$

$$\lambda = +\frac{5 \pm \sqrt{25 - 24}}{2} = \frac{5 \pm 1}{2} = 2, 3$$

$$\lambda = 1, \ (\lambda - 2)(\lambda - 3)$$

$$\boxed{\lambda = 1, 2, 3}$$

Now the eigen vectors of above eq^n can be calculated as

$$|A - \lambda I|x = 0$$

$$\begin{vmatrix} 1-\lambda & 0 & -1 \\ 1 & 2-\lambda & 1 \\ 2 & 2 & 3-\lambda \end{vmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

when $\lambda = 1$

Then

$$\begin{vmatrix} 0 & 0 & -1 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix} = 0$$

$z = 0$

$x + y + z = 0$

$2(x + y + z) = 0$

$x + y + z = 0$

$x + y = 0$

$x = -y$

$\frac{x}{1} = \frac{y}{-1}$ i.e. $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

||ly when $\lambda = 2$

$$\begin{vmatrix} -1 & 0 & -1 \\ 1 & 0 & 1 \\ 2 & 2 & 1 \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix} = 0$$

$-x + 0y - z = 0$

$x + 0y + z = 0$

$2x + 2y + z = 0$

[illegible] be calculated by using last [illegible] eq^n & applying crossmultiplication method